Performance Analysis of λ-MRC Decode and Forward Cooperation in Nakagami-m Fading for Arbitrary Parameters

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***Abstract*—In this paper, bit error rate (BER) expressions for the λ-MRC receiver for a decode and forward (DF) cooperative system are obtained for Nakagami-m fading, where m is not an integer. Previous results were available only for integer values of m. BER analysis is done by employing approximate statistics of a gamma conditionally gaussian (CG) random variable (RV) obtained through the Loskot-Prony approximation. Numerical results obtained using the analytical BER expressions are shown to closely follow the simulation results, despite the cumulative distribution function (CDF) of the gamma CGRV being a high signal to noise ratio (SNR) appoximation.**

***Index Terms*—BER, Gamma CG distribution, Nakagami-m fading**

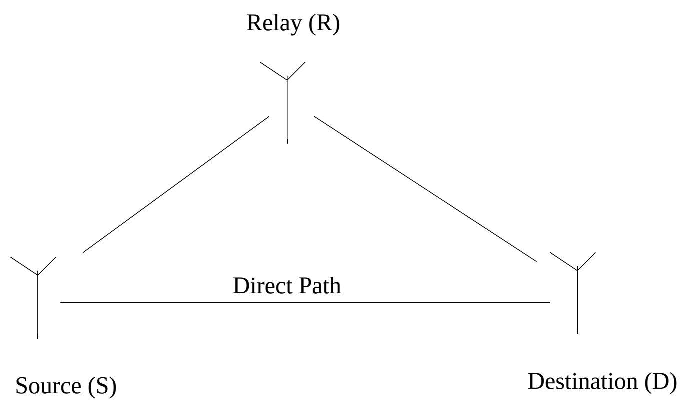

Fig. 1. Three node cooperative diversity system.

I. Introduction

BER expressions for DF cooperative systems, compared to amplify and forward (AF) cooperation, are difficult to evaluate, and hence there is considerable interest in finding analytical expressions for the BER for DF cooperative systems. Exact expressions for the BER for the piecewise linear (PL) combiner were first obtained for Rayleigh fading in [1] and [2] for noncoherent binary frequency shift keying (BFSK) and binary phase shift keying (BPSK) respectively. Results for the more general Nakagami-m fading were first obtained in [3] using the approach in [2] followed by a simpler approach in [4]. [4] also included BER expressions for the λ-MRC receiver proposed in [5].

One common feature of the above literature is the restriction of the Nakagami fading paramter m, to being an integer. To the best of our knowledge, there has not been any attempt to evalute the BER for popular DF receivers like the λ-MRC or PL combiner for noninteger values of m.

In this paper, we use a Loskot-Prony [6] approximation for the CDF of a gamma CGRV to obtain the BER for a λ-MRC cooperative system. This approximation for the CDF is known to be tight for high SNR. However, through numerical results, we show that the related expression for the BER obtained in this paper using this approxmiate CDF, exactly follows the simulation results.

In the beginning section, the system model is presented, followed by BER analysis. Numerical and simulation results are discussed next. Our conclusions are summarized in the final section, outlining the scope for future work.

II. System Model

The classic three node cooperative system in Figure 1 is considered, where, without loss of gen-

erality, h represents the Nakagami-m channel gain with fading figures m and Ω, E the transmit power at a node, x the transmitted symbol at a node, and the subscripts s and r the source and relay parameters respectively.

A. λ-MRC

The decision statistic for the λ-MRC receiver for BPSK modulation, is given by [4], [5]

$$X + \lambda Y \underset{-1}{\overset{1}{\gtrless}} 0, \quad 0 < \lambda \leq 1 \tag{1}$$

where $X \sim \mathcal{N}\left(a_s h_s^2, b_s h_s^2\right)$, $Y \sim \mathcal{N}\left(a_r h_r^2, b_r h_r^2\right)$ with $a_i = \frac{4E_i x_i}{N_0}, b_i = \frac{8E_i}{N_0}, c_i = \frac{m_i}{\Omega_i}, i \in \{s, r\}$.

$$p_{h_i^2}(x) = \frac{c_i^{m_i} x^{m_i - 1}}{\Gamma(m_i)} \exp\left(-c_i x\right), \quad x, c_i > 0, m_i \geq 0.5 \tag{2}$$

$h_i^2 \sim \mathcal{G}\left(c_i, m_i\right)$, where $\mathcal{G}$ denotes the *Gamma* distribution [8]. Assuming equal probability of the transmitted symbol $x_s = \{1, -1\}$, from (1), the average BER for a λ-MRC cooperative system can be expressed as

$$P_e = \sum_{x_r \in \{1,-1\}} \varepsilon^{\frac{1-x_r}{2}} (1-\varepsilon)^{\frac{1+x_r}{2}} P\left(X + \lambda Y < 0 | x_s = 1, x_r\right). \tag{3}$$

where ε is the BER for the S-R link.

III. BER Analysis for λ-MRC

From (3), we observe that the BER has to be computed separately for the case of correct and incorrect decision at the relay.

A. Correct Decision at Relay

The probability of error, given a correct decision at the relay, can be expressed as

$$\begin{aligned} P_{e|1} &= P\left(X + \lambda Y < 0 | x_s = 1, x_r = 1\right) \\ &= \int_{-\infty}^{\infty} F_X(-\lambda y) p_Y(y)\, dy \end{aligned} \tag{4}$$

To obtain the above, the statistics of X and Y are required. Since X and Y are conditionally Gaussian [3], their statistics are known for integer values of the Nakagami fading parameters m_i [3], [4]. Using this, the BER in (3) was obtained in [4]. For arbitrary m_i, while the exact PDF of X and Y is known (6), [4], an approximate expression for the CDF is available only for high SNR (7), using the Loskot-Prony approximation [6]. Due to space constraints, the proof of (7) is omitted in this paper. (4) can be expressed as

$$\begin{aligned} P_{e|1} = &\int_0^{\infty} F_X(-\lambda y) p_Y(y)\, dy \\ &+ \int_0^{\infty} F_X(\lambda y) p_Y(-y)\, dy \end{aligned} \tag{5}$$

Substituting $F_X, a_s > 0$ from (6) and (7) in the first integral in (5), we have

$$\int_0^{\infty} F_X(-\lambda y) p_Y(y)\, dy = \sum_{n=1}^{3} \frac{2a_n(\lambda)^{m_s}}{\Gamma(m_s)} \left(\frac{b_n}{\epsilon_s^2 \left(b_n + \frac{\kappa_s}{\epsilon_s}\right)}\right)^{m_s/2}$$

$Z \mid A \sim \mathcal{N}(aA, bA), b > 0$, $A \sim \mathcal{G}(c, m)$, $(a_1, a_2, a_3) = (0.168, 0.144, 0.002)$, $(b_1, b_2, b_3) = (0.876, 0.525, 0.603)$, $\epsilon = \frac{|a|}{c}, \kappa = \frac{b}{|a|}$, $K_{\cdot}(\cdot)$ is the modified Bessel function of the second kind [7] and $\gamma(\cdot, \cdot)$ is the lower incomplete gamma function [7].

$$p_Z(z) = \frac{2c^m e^{\frac{az}{b}}}{\Gamma(m)\sqrt{2\pi b}} \left(\frac{|z|}{\sqrt{a^2 + 2bc}}\right)^{m - \frac{1}{2}} K_{m - \frac{1}{2}}\left(\frac{|z|}{b}\sqrt{a^2 + 2bc}\right), \quad a > 0 \tag{6}$$

$$F_Z(z) \approx \begin{cases} \sum_{n=1}^3 \frac{2a_n(-z)^m e^{2b_n z/\kappa}}{\Gamma(m)} \left(\frac{b_n}{\epsilon^2\left(b_n + \frac{\kappa}{\epsilon}\right)}\right)^{m/2} K_m\left(-\frac{2z}{\kappa}\sqrt{b_n\left(b_n + \frac{\kappa}{\epsilon}\right)}\right) & a > 0, z \leq 0 \\ 1 - \frac{\gamma(m, \frac{-z}{\epsilon})}{\Gamma(m)} - \sum_{n=1}^3 \frac{2(-z)^m a_n e^{\frac{-2b_n z}{\kappa}}}{\Gamma(m)} \left(\frac{b_n}{\epsilon^2(b_n + \frac{\kappa}{\epsilon})}\right)^{\frac{m}{2}} K_m\left(\frac{-2z}{k}\sqrt{b_n(b_n + \frac{\kappa}{\epsilon})}\right) & a < 0, z \leq 0 \\ \frac{\gamma\left(m, \frac{z}{\epsilon}\right)}{\Gamma(m)} + \sum_{n=1}^3 \frac{2a_n z^m e^{2b_n z/\kappa}}{\Gamma(m)} \left(\frac{b_n}{\epsilon^2\left(b_n + \frac{\kappa}{\epsilon}\right)}\right)^{m/2} K_m\left(\frac{2z}{\kappa}\sqrt{b_n\left(b_n + \frac{\kappa}{\epsilon}\right)}\right) & a > 0, z > 0 \\ 1 - \sum_{n=1}^3 \frac{2(z)^m a_n e^{\frac{-2b_n z}{\kappa}}}{\Gamma(m)} \left(\frac{b_n}{\epsilon^2(b_n + \frac{\kappa}{\epsilon})}\right)^{\frac{m}{2}} K_m\left(\frac{2z}{\kappa}\sqrt{b_n(b_n + \frac{\kappa}{\epsilon})}\right) & a < 0, z > 0 \end{cases} \tag{7}$$

$$\times \frac{2c_r^{m_r}}{\Gamma(m_r)\sqrt{2\pi b_r}}\left(\frac{1}{\sqrt{a_r^2+2b_rc}}\right)^{m_r-\frac{1}{2}}$$
$$\times \int_0^\infty y^{m_s+m_r-\frac{1}{2}} e^{\frac{a_r y}{b_r}-\frac{2b_n\lambda y}{\kappa_s}} K_{m_s}\left(\frac{2\lambda y}{\kappa_s}\sqrt{b_n\left(b_n+\frac{\kappa_s}{\epsilon_s}\right)}\right)$$
$$\times K_{m_r-\frac{1}{2}}\left(\frac{y}{b_r}\sqrt{a_r^2+2b_rc_r}\right)dy \quad (8)$$

The above integral is of the form

$$\mathcal{I}_{m,n}(\alpha,\beta,\delta)=\int_0^\infty y^{m+n}e^{\alpha y}K_m(\beta y)K_n(\delta y)\,dy,$$
$$\{m,n,\beta,\delta\}>0. \quad (9)$$

This integral does not appear to be tabulated and is difficult to obtain in closed form. However, from (5), (6),(7), it is evident that the integral appears in the final expression for the BER and we will use (9) repeatedly in the following to represent integrals of the form in (8). The second integral in (5) can now be expressed as

$$\int_0^\infty F_X(\lambda y)p_Y(-y)\,dy$$
$$=\frac{1}{\Gamma(m_s)}\int_0^\infty \gamma\left(m_s,\frac{\lambda y}{\epsilon_s}\right)p_Y(-y)\,dy$$
$$+\sum_{n=1}^{3}\frac{2a_n(\lambda)^{m_s}}{\Gamma(m_s)}\left(\frac{b_n}{\epsilon_s^2\left(b_n+\frac{\kappa_s}{\epsilon_s}\right)}\right)^{m_s/2}$$
$$\times \frac{2c_r^{m_r}}{\Gamma(m_r)\sqrt{2\pi b_r}}\left(\frac{1}{\sqrt{a_r^2+2b_rc}}\right)^{m_r-\frac{1}{2}}$$
$$\times \mathcal{I}_{m_s,m_r-\frac{1}{2}}\left(-\frac{a_r}{b_r}+\frac{2b_n\lambda}{\kappa_s},\frac{2\lambda}{\kappa_s}\sqrt{b_n\left(b_n+\frac{\kappa_s}{\epsilon_s}\right)},\frac{1}{b_r}\sqrt{a_r^2+2b_rc_r}\right) \quad (10)$$

The first integral in (10) can be expressed using integration by parts as

$$\frac{1}{\Gamma(m_s)}\int_0^\infty \gamma\left(m_s,\frac{\lambda y}{\epsilon_s}\right)p_Y(-y)\,dy$$
$$=-\frac{1}{\Gamma(m_s)}\left[\left\{\gamma\left(m_s,\frac{\lambda y}{\epsilon_s}\right)F_Y(-y)\right\}_0^\infty\right.$$
$$\left.+\left(\frac{\lambda}{\epsilon_s}\right)^{m_s}\int_0^\infty y^{m_s-1}e^{-\frac{\lambda y}{\epsilon_s}}F_Y(-y)\,dy\right] \quad (11)$$

resulting in

$$\frac{1}{\Gamma(m_s)}\int_0^\infty \gamma\left(m_s,\frac{\lambda y}{\epsilon_s}\right)p_Y(-y)\,dy$$
$$=\frac{1}{\Gamma(m_s)}\left(\frac{\lambda}{\epsilon_s}\right)^{m_s}\int_0^\infty y^{m_s-1}e^{-\frac{\lambda y}{\epsilon_s}}F_Y(-y)\,dy$$
$$=\frac{1}{\Gamma(m_s)}\left(\frac{\lambda}{\epsilon_s}\right)^{m_s}\sum_{n=1}^{3}\frac{2a_n}{\Gamma(m_r)}\left(\frac{b_n}{\epsilon^2\left(b_n+\frac{\kappa_r}{\epsilon_r}\right)}\right)^{m_r/2}$$
$$\times\int_0^\infty y^{m_s+m_r-1}e^{-\left(\frac{\lambda}{\epsilon_s}+\frac{2b_n}{\kappa_r}\right)y}$$
$$\times K_{m_r}\left(\frac{2y}{\kappa_r}\sqrt{b_n\left(b_n+\frac{\kappa_r}{\epsilon_r}\right)}\right)dy \quad (12)$$

upon substituting for F_Y, $a_r>0$ from (7). From [9, (6.619.3)]

$$\int_0^\infty x^{\mu-1}e^{-\alpha x}K_\nu(\beta x)\,dx$$
$$=\frac{\sqrt{\pi}\,(2\beta)^\nu}{(\alpha+\beta)^{\mu+\nu}}\frac{\Gamma(\mu+\nu)\Gamma(\mu-\nu)}{\Gamma\left(\mu+\frac{1}{2}\right)}$$
$$\times {}_2F_1\left(\mu+\nu,\nu+\frac{1}{2};\mu+\frac{1}{2};\frac{\alpha-\beta}{\alpha+\beta}\right)$$
$$\mathrm{Re}\{\mu\}>|\mathrm{Re}\nu|,\ \mathrm{Re}(\alpha+\beta)>0. \quad (13)$$

Using (13) in (12), we obtain

$$\frac{1}{\Gamma(m_s)}\int_0^\infty \gamma\left(m_s,\frac{\lambda y}{\epsilon_s}\right)p_Y(-y)\,dy$$
$$=\frac{\Gamma(m_s+2m_r)}{\Gamma\left(m_s+m_r+\frac{1}{2}\right)}\left(\frac{\lambda}{\epsilon_s}\right)^{m_s}\sum_{n=1}^{3}\frac{2a_n}{\Gamma(m_r)}\left(\frac{b_n}{\epsilon^2\left(b_n+\frac{\kappa_r}{\epsilon_r}\right)}\right)^{m_r/2}$$
$$\times\frac{\sqrt{\pi}\left[\frac{4}{\kappa_r}\sqrt{b_n\left(b_n+\frac{\kappa_r}{\epsilon_r}\right)}\right]^{m_r}}{\left[\left(\frac{\lambda}{\epsilon_s}+\frac{2b_n}{\kappa_r}\right)+\frac{2}{\kappa_r}\sqrt{b_n\left(b_n+\frac{\kappa_r}{\epsilon_r}\right)}\right]^{m_s+2m_r}}$$
$${}_2F_1\left(\begin{matrix}m_s+2m_r,\ m_r+\frac{1}{2}\\ m_s+m_r+\frac{1}{2}\end{matrix};\frac{\left(\frac{\lambda}{\epsilon_s}+\frac{2b_n}{\kappa_r}\right)-\frac{2}{\kappa_r}\sqrt{b_n\left(b_n+\frac{\kappa_r}{\epsilon_r}\right)}}{\left(\frac{\lambda}{\epsilon_s}+\frac{2b_n}{\kappa_r}\right)+\frac{2}{\kappa_r}\sqrt{b_n\left(b_n+\frac{\kappa_r}{\epsilon_r}\right)}}\right), \quad (14)$$

Thus, from (4)-(14), we obtain

$$P(X+\lambda Y<0|x_s=1,x_r=1)=$$
$$\sum_{n=1}^{3}\frac{2a_n(\lambda)^{m_s}}{\Gamma(m_s)}\left(\frac{b_n}{\epsilon_s^2\left(b_n+\frac{\kappa_s}{\epsilon_s}\right)}\right)^{m_s/2}$$

$$\times \frac{2c_r^{m_r}}{\Gamma(m_r)\sqrt{2\pi b_r}}\left(\frac{1}{\sqrt{a_r^2+2b_r c}}\right)^{m_r-\frac{1}{2}}$$

$$\times\left\{\mathcal{I}_{m_s,m_r-\frac{1}{2}}\left(\frac{a_r}{b_r}-\frac{2b_n\lambda}{\kappa_s},\frac{2\lambda}{\kappa_s}\sqrt{b_n\left(b_n+\frac{\kappa_s}{\epsilon_s}\right)},\frac{1}{b_r}\sqrt{a_r^2+2b_r c_r}\right)\right.$$
$$\left.+\mathcal{I}_{m_s,m_r-\frac{1}{2}}\left(-\frac{a_r}{b_r}-\frac{2b_n\lambda}{\kappa_s},\frac{2\lambda}{\kappa_s}\sqrt{b_n\left(b_n+\frac{\kappa_s}{\epsilon_s}\right)},\frac{1}{b_r}\sqrt{a_r^2+2b_r c_r}\right)\right\}$$
$$+\frac{\Gamma(m_s+2m_r)}{\Gamma\left(m_s+m_r+\frac{1}{2}\right)}\left(\frac{\lambda}{\epsilon_s}\right)^{m_s}\sum_{n=1}^{3}\frac{2a_n}{\Gamma(m_r)}\left(\frac{b_n}{\epsilon^2\left(b_n+\frac{\kappa_r}{\epsilon_r}\right)}\right)^{m_r/2}$$
$$\times\frac{\sqrt{\pi}\left[\frac{4}{\kappa_r}\sqrt{b_n\left(b_n+\frac{\kappa_r}{\epsilon_r}\right)}\right]^{m_r}}{\left[\left(\frac{\lambda}{\epsilon_s}+\frac{2b_n}{\kappa_r}\right)+\frac{2}{\kappa_r}\sqrt{b_n\left(b_n+\frac{\kappa_r}{\epsilon_r}\right)}\right]^{m_s+2m_r}}$$
$$ {}_2F_1\left(\begin{matrix} m_s+2m_r,\ m_r+\frac{1}{2} \\ m_s+m_r+\frac{1}{2}\end{matrix};\frac{\left(\frac{\lambda}{\epsilon_s}+\frac{2b_n}{\kappa_r}\right)-\frac{2}{\kappa_r}\sqrt{b_n\left(b_n+\frac{\kappa_r}{\epsilon_r}\right)}}{\left(\frac{\lambda}{\epsilon_s}+\frac{2b_n}{\kappa_r}\right)+\frac{2}{\kappa_r}\sqrt{b_n\left(b_n+\frac{\kappa_r}{\epsilon_r}\right)}}\right), \quad (15)$$

B. Incorrect Decision at Relay

Given that an incorrect decision is made at the relay, the probability of error can be expressed as

$$\begin{aligned} P_{e|-1} &= \Pr\left(X+\lambda Y<0|x_s=1,x_r=-1\right)\\ &= \Pr\left(Y<\frac{-X}{\lambda}\middle| x_s=1,x_r=-1\right)\\ &= \int_{-\infty}^{\infty}F_Y\left(\frac{-x}{\lambda}\right)p_X(x)dx\\ &= \int_0^{\infty}F_Y\left(\frac{-x}{\lambda}\right)p_X(x)dx\\ &\quad+\int_0^{\infty}F_Y\left(\frac{x}{\lambda}\right)p_X(-x)dx \end{aligned} \quad (16)$$

The first integral in (16), upon substitution from (7) for $F_Y, a_r<0$ is

$$\begin{aligned}\int_0^{\infty}F_Y\left(\frac{-x}{\lambda}\right)p_X(x)dx &= \int_0^{\infty}p_X(x)dx\\ &-\frac{1}{\Gamma(m_r)}\int_0^{\infty}\gamma\left(m_r,\frac{x}{\lambda\epsilon_r}\right)p_X(x)dx\\ &-\sum_{n=1}^{3}\frac{2a_n}{\Gamma(m_r)}\left(\frac{b_n}{\epsilon_r^2\left(b_n+\frac{k_r}{\epsilon_r}\right)}\right)^{\frac{m_r}{2}}\left(\frac{1}{\lambda}\right)^{m_r}\\ &\times\frac{2c_s^{m_s}}{\Gamma(m_s)\sqrt{2\pi b_s}}\left(\frac{1}{\sqrt{a_s^2+2b_s c_s}}\right)^{m_s-\frac{1}{2}}\\ \times\int_0^{\infty}e^{x\left(\frac{a_s}{b_s}+\frac{2b_n}{\lambda\kappa_r}\right)}x^{m_r+m_s-\frac{1}{2}}&K_{m_r}\left(\frac{2x}{\lambda\kappa_r}\sqrt{b_n\left(b_n+\frac{\kappa_r}{\epsilon_r}\right)}\right)\\ &\times K_{m_s-\frac{1}{2}}\left(\frac{x}{b_s}\sqrt{(a_s^2+2b_s c_s)}\right)dx \end{aligned}\quad (17)$$

where the third integral in (17) is obtained after subsituting for $p_X, a_s>0$ from (6). The second integral in (17) can be expressed using the approach in (11) as

$$\begin{aligned}\frac{1}{\Gamma(m_r)}&\int_0^{\infty}\gamma\left(m_r,\frac{x}{\lambda\epsilon_r}\right)p_X(x)dx\\ &=1-\frac{1}{\Gamma(m_r)}\int_0^{\infty}\left(\frac{1}{\lambda\epsilon_r}\right)^{m_r}x^{m_r-1}e^{\frac{-x}{\lambda\epsilon_r}}F_X(x)dx\end{aligned}\quad (18)$$

Substituting $F_X, (a_s>0)$, from (7) in (18),

$$\begin{aligned}\frac{1}{\Gamma(m_r)}&\int_0^{\infty}\gamma\left(m_r,\frac{x}{\lambda\epsilon_r}\right)p_X(x)dx=1\\ &-\frac{1}{\Gamma(m_r)}\left(\frac{1}{\lambda\epsilon_r}\right)^{m_r}\left\{\int_0^{\infty}\frac{x^{m_r-1}e^{\frac{-x}{\lambda\epsilon_r}}\gamma(m_s,\frac{x}{\epsilon_s})}{\Gamma(m_s)}dx\right.\\ &+\int_0^{\infty}\sum_{n=1}^{3}\frac{2a_n x^{m_s}}{\Gamma(m_s)}e^{\frac{2b_n x}{\kappa_s}}\left(\frac{b_n}{\epsilon_s^2\left(b_n+\frac{k_s}{\epsilon_s}\right)}\right)^{\frac{m_s}{2}}\\ &\left.\times K_{m_s}\left(\frac{2x}{k_s}\sqrt{b_n\left(b_n+\frac{k_s}{\epsilon_s}\right)}\right)x^{m_r-1}e^{\frac{-x}{\lambda\epsilon_r}}dx\right\}\end{aligned}\quad (19)$$

The first integral in (19) can be expressed using [10, p. 138, (7)] as

$$\begin{aligned}\frac{1}{\Gamma(m_r)}&\left(\frac{\epsilon_s}{\lambda\epsilon_r}\right)^{m_r}\frac{1}{\Gamma(m_s)}\int_0^{\infty}t^{m_r-1}e^{\frac{-t\epsilon_s}{\lambda\epsilon_r}}\gamma(m_s,t)dt\\ &=\frac{1}{\Gamma(m_r)}\left(\frac{\epsilon_s}{\lambda\epsilon_r}\right)^{m_r}\frac{1}{\Gamma(m_s)}\frac{\Gamma(m_s+m_r)}{m_s\left(1+\frac{\epsilon_s}{\lambda\epsilon_r}\right)^{m_s+m_r}}\\ &\times{}_2F_1\left(1,m_s+m_r;m_s+1;\frac{\lambda\epsilon_r}{\lambda\epsilon_r+\epsilon_s}\right)\end{aligned}\quad (20)$$

The second integral in (19) can be expressed using (13) as

$$\frac{1}{\Gamma(m_r)}\left(\frac{1}{\lambda\epsilon_r}\right)^{m_r}\sum_{n=1}^{3}\frac{2a_n}{\Gamma(m_s)}\left(\frac{b_n}{\epsilon_s^2\left(b_n+\frac{k_s}{\epsilon_s}\right)}\right)^{\frac{m_s}{2}}$$
$$\int_0^{\infty}x^{m_s+m_r-1}e^{-x\left(\frac{1}{\lambda\epsilon_r}-\frac{2b_n}{k_s}\right)}K_{m_s}\left(\frac{2x}{k_s}\sqrt{b_n\left(b_n+\frac{k_s}{\epsilon_s}\right)}\right)$$
$$=\frac{\Gamma(2m_s+m_r)}{\Gamma(m_s+m_r+\frac{1}{2})}\left(\frac{1}{\lambda\epsilon_r}\right)^{m_r}\sum_{n=1}^{3}\frac{a_n}{\Gamma(m_s)}\left(\frac{b_n}{\epsilon_s^2\left(b_n+\frac{k_s}{\epsilon_s}\right)}\right)^{\frac{m_s}{2}}$$

$$\times \frac{\sqrt{\pi}\left(2 \times \frac{2}{k_s}\sqrt{b_n\left(b_n + \frac{\kappa_s}{\epsilon_s}\right)}\right)^{m_s}}{\left(\frac{1}{\lambda\epsilon_r} + \frac{2b_n}{\kappa_s} + \frac{2}{k_s}\sqrt{b_n\left(b_n + \frac{\kappa_s}{\epsilon_s}\right)}\right)^{2m_s+m_r}}$$

$$\times {}_2F_1\left(\begin{matrix} 2m_s + m_r, \; m_s + \frac{1}{2} \\ m_s + m_r + \frac{1}{2} \end{matrix}; \frac{\left(\frac{1}{\lambda\epsilon_r} - \frac{2b_n}{\kappa_s} - \frac{2}{k_s}\sqrt{b_n\left(b_n + \frac{\kappa_s}{\epsilon_s}\right)}\right)}{\left(\frac{1}{\lambda\epsilon_r} - \frac{2b_n}{\kappa_s} + \frac{2}{k_s}\sqrt{b_n\left(b_n + \frac{\kappa_s}{\epsilon_s}\right)}\right)}\right) \quad (21)$$

The third integral in (17) can be expressed using (9) as,

$$\sum_{n=1}^{3} \frac{2a_n}{\Gamma(m_r)}\left(\frac{b_n}{\epsilon_r^2\left(b_n + \frac{\kappa_r}{\epsilon_r}\right)}\right)^{\frac{m_r}{2}}\left(\frac{1}{\lambda}\right)^{m_r}$$

$$\times \frac{2c_s^{m_s}}{\Gamma(m_s)\sqrt{2\pi b_s}}\left(\frac{1}{\sqrt{a_s^2 + 2b_s c_s}}\right)^{m_s - \frac{1}{2}}$$

$$\times \mathcal{I}_{m_r, m_s - \frac{1}{2}}\left(\frac{a_s}{b_s} + \frac{2b_n}{\lambda\kappa_r}, \frac{2}{\lambda k_r}\sqrt{b_n\left(b_n + \frac{k_r}{\epsilon_r}\right)}, \frac{1}{b_s}\sqrt{\left(a_s^2 + 2b_s c_s\right)}\right) \quad (22)$$

With this, all integrals in (17) are evaluated. The second integral in (16) can be expressed as

$$\int_0^\infty F_Y\left(\frac{x}{\lambda}\right) p_X(-x)dx = \int_0^\infty p_X(-x)dx$$

$$- \sum_{n=1}^{3} \frac{2a_n}{\Gamma(m_r)}\left(\frac{b_n}{\epsilon_r^2\left(b_n + \frac{k_r}{\epsilon_r}\right)}\right)^{\frac{m_r}{2}}\left(\frac{1}{\lambda}\right)^{m_r}$$

$$\times \frac{2c_s^{m_s}}{\Gamma(m_s)\sqrt{2\pi b_s}}\left(\frac{1}{\sqrt{a_s^2 + 2b_s c_s}}\right)^{m_s - \frac{1}{2}}$$

$$\times \int_0^\infty e^{x\left(\frac{-a_s}{b_s} + \frac{-2b_n}{\lambda k_r}\right)} x^{m_r + m_s - \frac{1}{2}} K_{m_r}\left(\frac{2x}{\lambda\kappa_r}\sqrt{b_n\left(b_n + \frac{\kappa_r}{\epsilon_r}\right)}\right)$$

$$\times K_{m_s - \frac{1}{2}}\left(\frac{x}{b_s}\sqrt{(a_s^2 + 2b_s c_s)}\right)dx \quad (23)$$

From (9), the second integral in (23) is obtained as

$$\sum_{n=1}^{3} \frac{2a_n}{\Gamma(m_r)}\left(\frac{b_n}{\epsilon_r^2\left(b_n + \frac{k_r}{\epsilon_r}\right)}\right)^{\frac{m_r}{2}}\left(\frac{1}{\lambda}\right)^{m_r}$$

$$\times \frac{2c_s^{m_s}}{\Gamma(m_s)\sqrt{2\pi b_s}}\left(\frac{1}{\sqrt{a_s^2 + 2b_s c_s}}\right)^{m_s - \frac{1}{2}}$$

$$\times \mathcal{I}_{m_r, m_s - \frac{1}{2}}\left(-\frac{a_s}{b_s} - \frac{2b_n}{\lambda\kappa_r}, \frac{2}{\lambda k_r}\sqrt{b_n\left(b_n + \frac{\kappa_r}{\epsilon_r}\right)}, \frac{1}{b_s}\sqrt{\left(a_s^2 + 2b_s c_s\right)}\right) \quad (24)$$

From (16)-(24), we obtain

$$P(X + \lambda Y < 0 | x_s = 1, x_r = -1)$$

$$= -\sum_{n=1}^{3} \frac{2a_n}{\Gamma(m_r)}\left(\frac{b_n}{\epsilon_r^2\left(b_n + \frac{\kappa_r}{\epsilon_r}\right)}\right)^{\frac{m_r}{2}}$$

$$\times \left(\frac{1}{\lambda}\right)^{m_r} \frac{2c_s^{m_s}}{\Gamma(m_s)\sqrt{2\pi b_s}}\left(\frac{1}{\sqrt{a_s^2 + 2b_s c_s}}\right)^{m_s - \frac{1}{2}} \times$$

$$\Bigg\{\mathcal{I}_{m_r, m_s - \frac{1}{2}}\left(-\frac{a_s}{b_s} - \frac{2b_n}{\lambda\kappa_r}, \frac{2}{\lambda k_r}\sqrt{b_n\left(b_n + \frac{\kappa_r}{\epsilon_r}\right)}, \frac{1}{b_s}\sqrt{\left(a_s^2 + 2b_s c_s\right)}\right)$$

$$+ \mathcal{I}_{m_r, m_s - \frac{1}{2}}\left(\frac{a_s}{b_s} + \frac{2b_n}{\lambda\kappa_r}, \frac{2}{\lambda k_r}\sqrt{b_n\left(b_n + \frac{\kappa_r}{\epsilon_r}\right)}, \frac{1}{b_s}\sqrt{\left(a_s^2 + 2b_s c_s\right)}\right)\Bigg\}$$

$$+ \frac{1}{\Gamma(m_r)}\left(\frac{1}{\lambda\epsilon_r}\right)^{m_r} \sum_{n=1}^{3} \frac{a_n}{\Gamma(m_s)}\left(\frac{b_n}{\epsilon_s^2\left(b_n + \frac{k_s}{\epsilon_s}\right)}\right)^{\frac{m_s}{2}}$$

$$\times \frac{\sqrt{\pi}\left(2 \times \frac{2}{k_s}\sqrt{b_n\left(b_n + \frac{\kappa_s}{\epsilon_s}\right)}\right)^{m_s}}{\left(\frac{1}{\lambda\epsilon_r} + \frac{2b_n}{\kappa_s} + \frac{2}{k_s}\sqrt{b_n\left(b_n + \frac{\kappa_s}{\epsilon_s}\right)}\right)^{2m_s+m_r}}$$

$$\times \frac{\Gamma(2m_s + m_r)\Gamma(m_r)}{\Gamma(m_s + m_r + \frac{1}{2})}$$

$$\times {}_2F_1\left(\begin{matrix} 2m_s + m_r, \; m_s + \frac{1}{2} \\ m_s + m_r + \frac{1}{2} \end{matrix}; \frac{\left(\frac{1}{\lambda\epsilon_r} - \frac{2b_n}{\kappa_s} - \frac{2}{k_s}\sqrt{b_n\left(b_n + \frac{\kappa_s}{\epsilon_s}\right)}\right)}{\left(\frac{1}{\lambda\epsilon_r} - \frac{2b_n}{\kappa_s} + \frac{2}{k_s}\sqrt{b_n\left(b_n + \frac{\kappa_s}{\epsilon_s}\right)}\right)}\right)$$

$$+ \frac{1}{\Gamma(m_r)}\left(\frac{\epsilon_s}{\lambda\epsilon_r}\right)^{m_r} \frac{1}{\Gamma(m_s)} \frac{\Gamma(m_s + m_r)}{m_s\left(1 + \frac{\epsilon_s}{\lambda\epsilon_r}\right)^{m_s + m_r}}$$

$$\times {}_2F_1\left(1, m_s + m_r; m_s + 1; \frac{1}{1 + \frac{\epsilon_s}{\lambda\epsilon_r}}\right) \quad (25)$$

Substituting (15) and (25) in (3), we obtain the final expression for the BER.

IV. Results

In Figure 2, the analytical and simulated BER are plotted with respect to the average SNR for the S-D link. For convenience, we have chosen $E_r = E_s$, i.e. the source and relay transmit with equal power. (15) and (25) are used to compute the analytical BER using (3) for two cases, $\lambda = 0.5$ and $\lambda = 1$. As we can see, there is an excellent match between the simulation and analytical results, validating the expressions derived in the paper. Note that the Nakagami fading parameters are not integers.

Figure 3 provides some interesting insights into the diversity order for λ-MRC cooperation. Firstly,

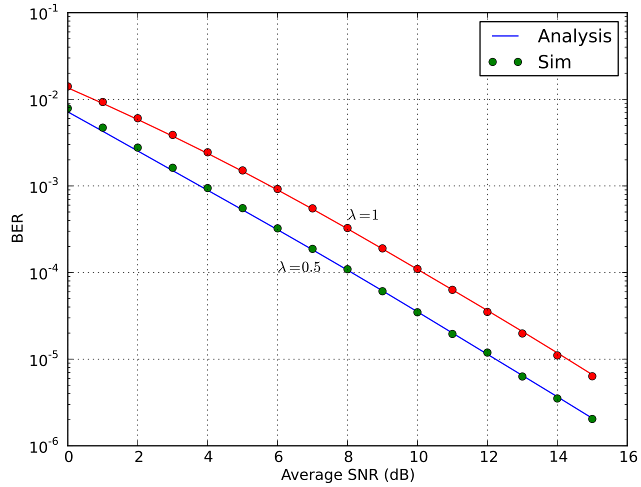

Fig. 2. Comparison of the simulation and analytical results for $m = 3.7, m_s = 2.6, m_r = 2.6$. Both match perfectly.

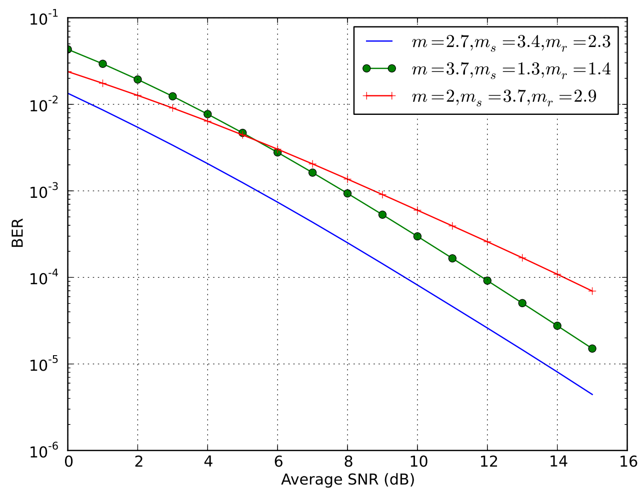

Fig. 3. Analytical BER plots for $\lambda = 1$. Slopes for the lower two curves almost identical at high SNR indicating a similar diversity order.

we note that the middle and bottom curves in Figure 3 have the same slope at high SNR, indicating the same diversity order. We note that $m_s + m_r = 2.7$ for the middle curve is exactly equal to $m = 2.7$ for the bottom curve. This validates the result in [11] where the diversity order was shown to be $\min(m, m_s + m_r)$ when $\lambda = 1$. Note that the top curve has a diversity order $2 < 2.7$ and its slope is less compared to that of the other two curves, at high SNR.

V. Conclusions and future work

In this paper, we have obtained a close but approximate expression for the BER for the λ-MRC-DF cooperative system. The final expression contains only one integral in terms of simple, well defined functions. Numerical results obtained using this expression match exactly with the actual simulation results, indicating the usefulness of this work. A closed form expression for the integral is a work in progress. Based on the techniques employed in this paper, it should be possible to find similar expressions for the BER for the superior PL combiner, and will be addressed in future work.

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